

Giant Steps in Cefalù

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Abstract. *Giant steps* is a technique to accelerate Monte Carlo radiative transfer in optically-thick cells (which are isotropic and homogeneous in matter properties and into which astrophysical atmospheres are divided) by greatly reducing the number of Monte Carlo steps needed to propagate photon packets through such cells. In an optically-thick cell, packets starting from any point (which can be regarded a point source) well away from the cell wall act essentially as packets diffusing from the point source in an infinite, isotropic, homogeneous atmosphere. One can replace many ordinary Monte Carlo steps that a packet diffusing from the point source takes by a randomly directed giant step whose length is slightly less than the distance to the nearest cell wall point from the point source. The giant step is assigned a time duration equal to the time for the RMS radius for a burst of packets diffusing from the point source to have reached the giant step length. We call assigning giant-step time durations this way RMS-radius (RMSR) synchronization. Propagating packets by series of giant steps in giant-steps random walks in the interiors of optically-thick cells constitutes the technique of *giant steps*. *Giant steps* effectively replaces the exact diffusion treatment of ordinary Monte Carlo radiative transfer in optically-thick cells by an approximate diffusion treatment. In this paper, we describe the basic idea of *giant steps* and report demonstration giant-steps flux calculations for the grey atmosphere. Speed-up factors of order 100 are obtained relative to ordinary Monte Carlo radiative transfer. In practical applications, speed-up factors of order ten [1] and perhaps more are possible. The speed-up factor is likely to be significantly application-dependent and there is a trade-off between speed-up and accuracy. This paper and past work [1] suggest that giant-steps error can probably be kept to a few percent by using sufficiently large boundary-layer optical depths while still maintaining large speed-up factors. Thus, *giant steps* can be characterized as a moderate accuracy radiative transfer technique. For many applications, the loss of some accuracy may be a tolerable price to pay for the speed-ups gained by using *giant steps*.

INTRODUCTION

Monte Carlo radiative transfer simulates radiation transfer by propagating photon packets (which represent aggregations of photons) through model atmospheres. The atmospheres are usually divided into cells that are isotropic and homogeneous in matter properties: e.g., slab cells that are infinite in 2 dimensions for plane-parallel atmospheres or concentric spherical shells for spherically symmetric atmospheres. From the packet-matter interactions, one can calculate cell thermal states and from the packets that escape the atmosphere, the emergent flux and spectrum. Since there are almost always vastly fewer packets than actual photons, one is doing a statistical sampling to determine results and these results will have errors due to statistical fluctuations. The statistical errors can be reduced by increasing the number of packets.

Monte Carlo radiative transfer has powerful features. Packet-matter interactions and complex atmospheres in multi-dimensions can usually be treated straightforwardly with

great physical realism. Thus, one can avoid many of the approximations or complex treatments needed for other radiative transfer techniques. Monte Carlo radiative transfer codes are often easy to develop in comparison to other radiative transfer codes. Monte Carlo radiative transfer is embarrassingly parallelizable in that one can propagate as many packets through an atmosphere simultaneously as one has processors: the effects of the packets merely have to be summed at the end of the propagation calculation. Monte Carlo radiative transfer gives robust convergence in self-consistent atmosphere calculations where an atmosphere solution is iterated to convergence via alternating radiative transfer and thermal state calculations [2, 3].

Monte Carlo radiative transfer also has severe disadvantages relative to other radiative transfer techniques. In doing Monte Carlo radiative transfer, it can be CPU time expensive to reduce the statistical errors to an acceptable level. Another (but related) problem is that Monte Carlo radiative transfer can be very slow for optically-thick atmospheres which in the deep interior will usually be divided into optically-thick cells. The slowness is caused, of course, by having to propagate each packet through all its many interactions as it traverses the optically-thick cells. The details of a packet's propagation are not of interest, but in the ordinary Monte Carlo treatment one must calculate them nevertheless.

Can anything be done about the slow propagation in optically-thick cells? Yes. Take giant steps.

THE BASIC IDEA OF GIANT STEPS

While mulling on Monte Carlo radiative transfer on the night of 2006 February 13, one of us (D.J.J.) had a eureka moment. Consider a packet (he said to himself) in an optically-thick cell of a pure-isotropic-scattering atmosphere just after a scattering event. The packet will random walk a long way in the cell and will always be in a random direction relative to the start point: as long as the packet does not leave the cell, its motion is just as if it were in an infinite, isotropic, homogeneous atmosphere. Why not just “giant-step” the packet from the start point in a random direction as far as one can go in the cell consistent with the packet behaving as if in an infinite, isotropic, homogeneous atmosphere. For the consistency, the giant-step length should not exceed (and perhaps be a bit less (see below)) than the distance to the nearest cell wall point: but note the giant step is in a random direction, not to the nearest cell wall point. After one giant step, take another giant step in a random direction, and so on in a giant-steps random walk. With this random walk, the packet is propagated a long way quickly inside the cell: there should be a great speed-up relative to ordinary Monte Carlo propagation.

Of course, how does the packet get out of the cell and if the packet is less than optical depth 1 from the cell wall, the giant steps turn into baby steps. Ah, change to ordinary Monte Carlo propagation when one is close to the cell wall: ordinary Monte Carlo steps have variable length determined by a cumulative probability distribution and a random direction. The ordinary Monte Carlo steps allow the packet to leave the cell or re-enter the giant-steps region (the interior region of the cell where only giant steps can be started.) It turns out that one needs a boundary layer (of optical depth τ_{boundary}) consisting of an ordinary-step layer (where only ordinary steps are taken) of optical

A photon packet on a giant-steps random walk through a pure-isotropic-scattering atmosphere

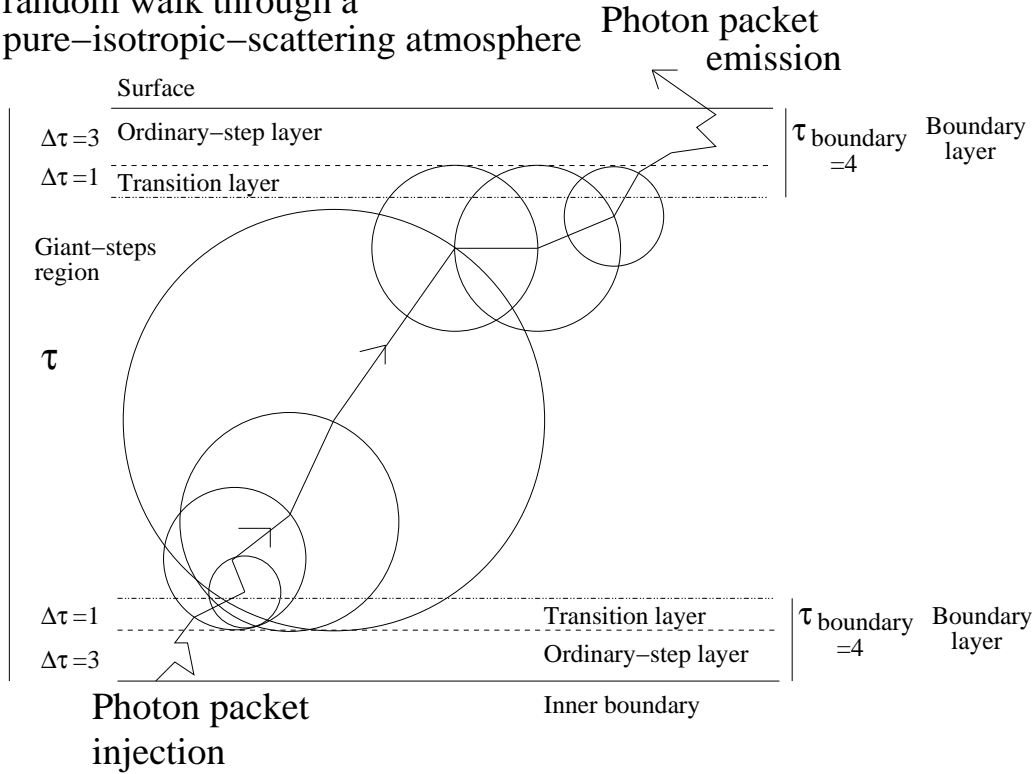


FIGURE 1. A cartoon of a giant-steps random walk in a plane-parallel, pure-isotropic-scattering atmosphere consisting of one cell of optical depth τ . The circles represent the spheres that are the loci formed from the giant-step endpoints possible for each start point in the giant-steps region. A giant step is taken in a random direction to the sphere from a start point.

depth a few adjacent to the cell wall and transition layer of optical depth 1 adjacent to the ordinary-step layer. Only ordinary steps are started in the boundary layer, but giant steps penetrate into the transition layer from the giant-steps region. With the boundary-layer setup, the giant-step length is determined by the distance to the nearest ordinary-step layer point, not as in first conception to the nearest cell wall point. Since the transition layer has optical depth 1, there are no baby steps. In fact, the ordinary-step layer can be shrunk to zero width, but for high accuracy it should have an optical depth of a few (see the next section). Figure 1 shows a cartoon of a giant-steps random walk through a plane-parallel, pure-isotropic-scattering atmosphere consisting of one cell.

Whenever one has a smart idea there are really only two possibilities: (a) one is wrong; (b) someone has thought of it before. *Giant steps* (as we call the technique described above) turned out to be case (b) when another of us (P.A.M.) admitted to having used *giant steps* (without calling it that) surreptitiously in supernova lightcurve calculations [1]. A full presentation of *giant steps* with discussion of how it applies to general atmospheres including, of course, multi-frequency treatment and energy deposition (which is needed for self-consistent atmosphere solutions) is given by [4]. Here we will elaborate just a bit on the description of *giant steps* given above.

Some thought (aided by some calculations) shows that *giant steps* is approximating packet diffusion in a cell rather than treating that diffusion exactly as in ordinary Monte Carlo radiative transfer. Consider two isotropic-scattering packets diffusing in an infinite, isotropic, homogeneous atmosphere. One packet starts diffusing from a point and the other packet starts diffusing from giant-step length away in a random direction starting an appropriate time duration later. The packets will have approximately the same average behavior. This can be seen by imagining each packet as part of a isotropic packet burst: the first from a point source; the second from a sphere source of radius the giant-step length. The behaviors of the bursts converge with time. This convergence is optimum if the time duration assigned to the giant step is the time for the root-mean-square (RMS) radius of the point-source burst to reach the giant-step length. We call setting the giant-step time durations this way RMS-radius (RMSR) synchronization. RMSR synchronization is the optimum synchronization for *giant steps* [4]. A key aspect of RMSR synchronization is that the RMS radii of point- and sphere-source bursts are always equal after the giant-step time duration. The point- and sphere-source bursts give a heuristic picture that allows one to understand *giant steps*. In actual calculations, one packet at a time (at least on one processor) is propagated through its whole trajectory in the atmosphere and the packets take series of giant steps in optically-thick cells.

DEMONSTRATION GIANT-STEPS FLUX CALCULATIONS

Counting giant-step time durations is essential in time-dependent giant-steps calculations and in any giant-steps calculations where energy deposition needs to be done (i.e., in any self-consistent atmosphere solution calculations). However, for just propagating packets through the grey atmosphere (a time-independent, isotropic-scattering, semi-infinite, plane-parallel atmosphere with frequency-independent opacity and which can be treated as a pure-isotropic-scattering atmosphere with effectively a single frequency: [e.g., 5, p. 53–76]) in order to get the emergent flux, time durations are not counted and one just does giant-step propagation of packets as described in the last section and as illustrated in Fig. 1. We have done such giant-steps flux calculations for the grey atmosphere with optical depth 1000 (and inner boundary treated in the diffusion approximation) treated as one cell and with 10^8 packets injected at the inner boundary. From the exact analytic solution of the grey atmosphere [e.g., 5, p. 71–73], we know that only about 1.3×10^5 packets will escape from the surface of the atmosphere: the others are absorbed at the inner boundary. As an example result, we consider the angular distribution of the emergent radiation field. Because it is convenient to evaluate in a Monte Carlo calculation, we have chosen to represent the angular distribution by relative partial astrophysical fluxes given by

$$\Delta f_i = 2 \int_{\mu_{i-1}}^{\mu_i} \frac{I(\mu)}{F} \mu d\mu, \quad (1)$$

where μ is the direction cosine for the angle from the normal to the surface, i is an index (running 1, 2, ...), $\mu_i - \mu_{i-1}$ is the μ interval for which Δf_i is evaluated, and $I(\mu)/F$ is the specific intensity divided by the astrophysical flux (i.e., the limb-darkening law). We

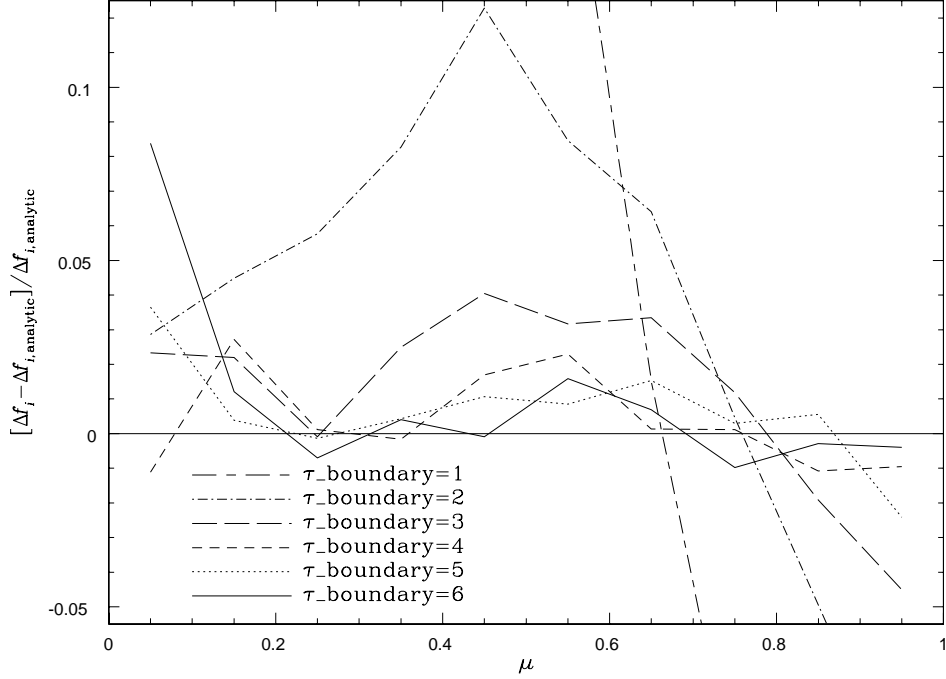


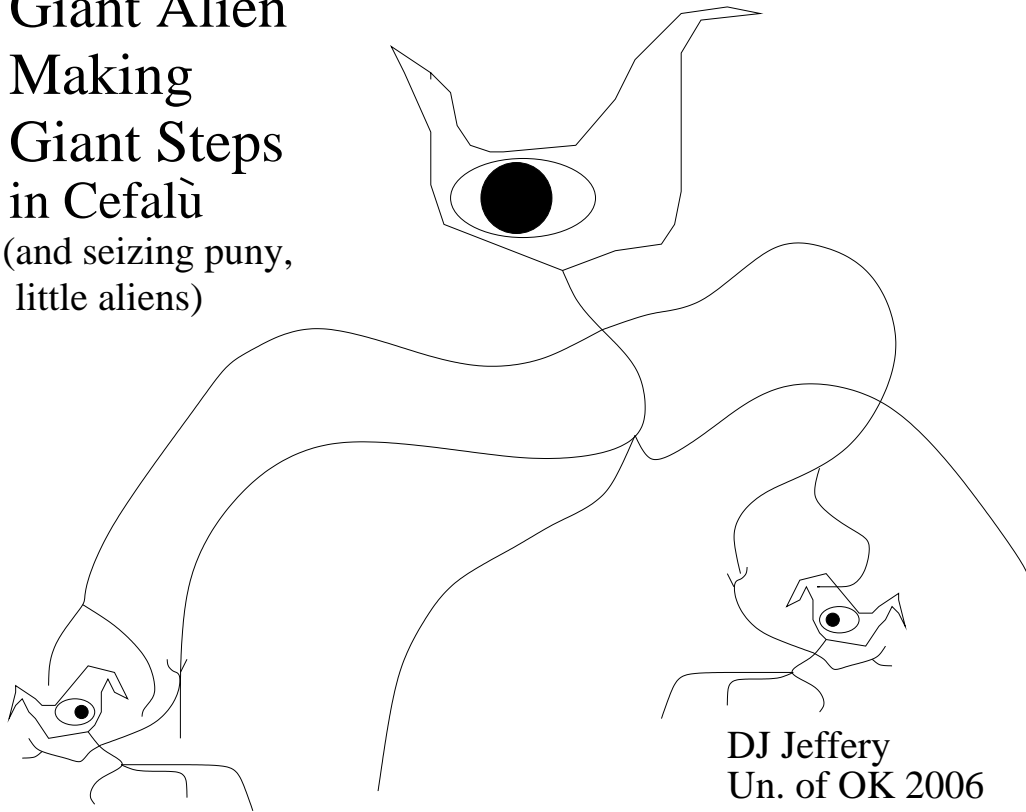
FIGURE 2. Relative deviations for Δf_i indicated at μ -interval midpoints by connecting lines.

evaluated Δf_i for ten μ intervals of 0.1 spanning the full range for emergent flux from $\mu = 0$ to $\mu = 1$ for a set of giant-steps calculations with varied boundary-layer optical depth τ_{boundary} . To test for accuracy, we evaluated the relative deviations of the giant-steps Δf_i values from ones calculated from the exact analytic limb-darkening law for the grey atmosphere. (The exact analytic limb-darkening law formula must be evaluated numerically: tabulations are given by, e.g., [5, p. 72] and [6, p. 135].) In Fig. 2, we have indicated these relative deviations at the μ -interval midpoints by connecting the values by lines: this is clearer than plotting multiple sets of data by data points or in a histogram.

From Fig. 2, we see that the accuracy is poor for $\tau_{\text{boundary}} \leq 2$ for which some relative deviations are off the plot. The maximum absolute value of a relative deviation is 1.29 at $\mu = 0.05$ for $\tau_{\text{boundary}} = 1$. Note, however, that the net flux for $\tau_{\text{boundary}} = 1$ is in error by only 3% [4]: thus, it is only the angular distribution that is rather inaccurate. The accuracy of the giant-steps calculations generally improves as τ_{boundary} is increased up to about $\tau_{\text{boundary}} = 6$ where error is a few percent or less. For higher values of τ_{boundary} , there is little significant improvement since the systematic error has become smaller than the Monte Carlo statistical error. The statistical error, in fact, increases as i decreases because the number of escaping packets per μ coordinate decreases as μ goes to 0. This explains the peak at $\mu = 0.05$ for $\tau_{\text{boundary}} = 6$: the peak is only a 2σ deviation, in fact.

The giant-steps calculations are much faster than the counterpart ordinary Monte Carlo calculations. The speed-up factors range from ~ 250 for $\tau_{\text{boundary}} = 1$ to ~ 70 for $\tau_{\text{boundary}} = 6$. Obviously, the speed-up factor decreases as the boundary layer (where ordinary steps are taken) increases.

Giant Alien Making Giant Steps in Cefalù (and seizing puny, little aliens)



DJ Jeffery
Un. of OK 2006

FIGURE 3. Giant Alien making giant steps in Cefalù.

Reader, conclusions are given in the abstract. We have run out of room here.

ACKNOWLEDGMENTS

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Since Hale Bradt of the scientific organizing committee asked participants for a personal image in the talks and we had no images at all in Cefalù, we have included Fig. 3 which illustrates the more fearful aspects of *giant steps*.

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